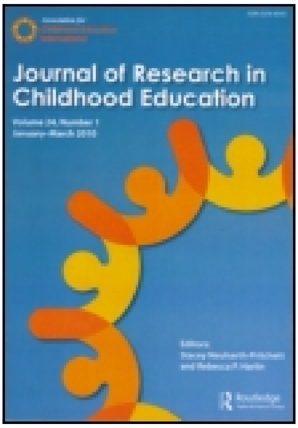

Journal of Research in Childhood Education

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/ujrc20

Pedagogical Content Knowledge for Preschool Mathematics: Construct Validity of a New Teacher Interview

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Available online: 19 Jun 2012

Journal of Research in Childhood Education, 26: 291–307, 2012

ISSN: 0256-8543 print / 2150-2641 online
DOI: 10.1080/02568543.2012.685123

Pedagogical Content Knowledge for Preschool Mathematics: Construct Validity of a New Teacher Interview

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This study examines the construct validity of a new teacher interview designed to assess teachers' pedagogical content knowledge (PCK) for preschool mathematics. PCK describes the subject matter knowledge a teacher needs for effective teaching. Data from 22 teachers and 113 Head Start children in a large midwestern city in the United States were used to test predictive relationships between the PCK interview and two dependent variables: good preschool math teaching practices and improved child learning outcomes. Using a method from previous published work, frequency of math-related language is assessed as a proxy for good preschool math teaching. Changes in children's mathematical achievement were measured from fall to spring within a single school year using the Test of Early Mathematics Ability (3rd ed.). Analysis by hierarchical linear modeling found significant positive relationships between scores on the new measure and both variables, suggesting that the PCK interview adequately represents the knowledge needed for effective teaching of preschool mathematics. A theoretical framework for the interview's construction is provided.

Keywords: early childhood mathematics, preschool, effective teaching, mathematics education, measurement, language

In 1988, Deborah Ball's dissertation demonstrated the importance of assessing teachers' pedagogical content knowledge (PCK) for teaching mathematics. Ball took Shulman's (1986) idea—that a particular kind of content knowledge is particularly suited to teaching—and used it as the basis for a mathematics interview. In it, teachers were presented with elementary classroom scenarios in which a student misunderstood a mathematics problem and then were asked how they would respond. Teachers with good PCK for teaching mathematics, Ball reasoned, would come up with mathematical representations that clarify meaning and guide thinking. Ball found that few teachers in the study had adequate scenario responses; some teacher responses were simply incorrect. More recently, this work has been transformed from an interview to a survey (see Hill, Schilling, & Ball, 2004) that assesses teachers' mathematical knowledge for teaching. Stronger teacher performance on the survey instrument has been found to predict growth in students' mathematics achievement (Hill, Rowan, & Ball, 2005). This work

Submitted October 17, 2010; accepted May 24, 2011.
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on PCK accomplished two important things for mathematics education: first, it provided clear evidence that U.S. teachers' PCK for mathematics teaching was lacking and second, it suggested the types of content and pedagogy that might constitute good PCK in mathematics, providing clear direction for elementary teacher education.

Unfortunately, Ball's interview and subsequent work related to PCK for teaching elementary mathematics offer little to those of us wishing to better understand preschool mathematics and its effective teaching because of profound differences in the knowledge bases of mathematics at these two developmental levels. The survey instrument described above is focused explicitly on elementary-level mathematics; it specifically gauges "proficiency at providing students with mathematical explanations and representation and working with unusual solution methods" (Hill et al., 2005, p. 387). Elementary mathematics uses written notation to support mathematical thinking, enabling the use of longer strings of procedures. This makes the provision of real-world representations and the analysis of different procedures—the items this survey is designed to measure—key to helping elementary schoolchildren develop a meaningful understanding of arithmetic.

Preschool children are active mathematics learners but are not ready to understand, let alone use, written arithmetic. Instead, 3- and 4-year-olds are primed to notice and explore the quantitative relationships in the world around them, and to begin using language, pictures, models, and other forms of representation to crystallize information about pattern, shape, space, and number. Because of this, the preschool instructional task shifts from creating a connected, conceptual understanding of new mathematical procedures, such as "moving the decimal over one place," to helping young children recognize, name, and experiment with the mathematics in their classroom environment (e.g., Copley, 2010; Ginsburg, Lee, & Boyd, 2008). These particular preschool pedagogical tasks require a knowledge base distinct from the one that previous work has helped to identify.

Meanwhile, a large body of literature indicates that high-quality mathematics education for young children builds a strong foundation for future mathematics learning. Through challenging and engaging early math education, young children acquire such important mathematical concepts as number sense, shape, measurement, and logical reasoning abilities. Early mathematics understanding significantly predicts school achievement in later years (Duncan et al., 2007), and early intervention specifically focused on mathematics has been shown to have broad positive effects on student learning (Fuson, Smith, & Lo Cicero, 1997). Clearly, early mathematics should not be dismissed as an important educational opportunity.

Although the education literature indicates that teacher quality is the most effective predictor of student achievement (Darling-Hammond, 2000; Just for the Kids and The Southeast Center for Teaching Quality, 2002; National Commission on Teaching & America's Future, 1996; Rice, 2003), the quality of mathematics teaching at the preschool level is extremely variable (Copple, 2004). A joint position statement on preschool math by the National Association for the Education of Young Children and the National Council of Teachers of Mathematics (NAEYC; 2005) notes the general lack of good teacher preparation in mathematics. Describing early childhood educators, Copley (2010) noted "to them, mathematics is a difficult subject to teach and one area that they often ignore except for counting and simple arithmetic" (p. 402). Early childhood teachers need a better understanding of what constitutes preschool mathematics and how to help young children construct that knowledge.

In an attempt to illuminate what preschool mathematics is and describe its basic elements for effective teaching, the first author designed a new teacher interview to measure PCK for preschool mathematics. To assess the interview's validity, we implemented it with a sample of Head Start teachers and children in a large midwestern city. The construct validity of a new measure is notoriously difficult to assess when the measure is the first of its kind (Strauss, 1987). In this case, the analysis assumes that good PCK will be associated with effective teaching practices and positive child outcomes. Accordingly, the interview results were evaluated as predictors of teacher observation data and change scores in the mathematics achievement of their students. This article describes the development of the interview, provides a theoretical justification of its design, and reports on its association to good preschool mathematics teaching practices and gains in children's mathematics learning.

CONSTRUCTION, FACE VALIDITY, PILOTING, AND INTERNAL RELIABILITY OF THE INTERVIEW

The interview (entire text is available by contacting editorial@acei.org) presents teachers with two classroom-based scenarios: one in the dramatic play area and one in the block corner. First, teachers read the scenarios to themselves and then the researcher reads them aloud; finally, the researcher asks the teacher a set of questions about each scenario. Scenario 1 is presented below as an illustration.

> Brittany and Jacob are playing in the dramatic play area and want to put their five babies to bed. There are no doll beds, so they make "cribs" out of three shoeboxes. Jacob says, "But there aren't enough cribs." Brittany responds, "These babies are younger," picking out the three babies with no hair and setting them near the shoeboxes. She picks up the two babies with thick hair, says, "These babies don't need to nap anymore," and sets them aside. Jacob says, "OK, but this baby needs the most room" and puts the biggest bald baby into the biggest shoebox. Brittany watches him and then puts the medium-sized baby in the medium-sized shoebox and the smallest bald baby in the smallest shoebox. Jacob says, "Now go to sleep, babies."

Questions follow, such as, "What kinds of mathematics do you see in this play?," "Where in the scenario do you see that math?," and "What might you say to help the children also see that math?"

Called the Preschool Mathematics PCK Interview (PM-PCK Interview), the instrument is designed to assess preschool teachers' pedagogical content knowledge in mathematics. Guided by the *Principles and Standards for School Mathematics* developed by the National Council of Teachers of Mathematics (NCTM; 2000), the interview scenarios cover a range of early mathematics concepts and skills through the mention of specific materials, the comments children make during the scenario, or the problems that children encounter and actions they take. For example, the small, medium, and large relationships of babies and shoeboxes might be seen as a repeating pattern, whereas using a shoebox for a crib utilizes three-dimensional geometric thinking. Recognizing that there "aren't enough cribs" requires one-to-one correspondence, which is foundational to number sense. When Brittany solves this dilemma by putting aside the two babies who have hair, she sorts a single set (babies) into two sets (bald babies and babies with hair). Measurement occurs when Jacob determines which baby needs the most room, and which shoebox offers it. In the interview, teachers who can see more mathematics in such play and generate comments that encourage its elaboration score more points.

A small group of experts in quantitative development and preschool pedagogy assessed the face validity of the interview and contributed possible teacher responses. The list was used to construct response rubrics for each interview question, which allow interviewers to check off key items as they are mentioned by teachers. After piloting among three preschool teachers, response rubrics were revised and resubmitted to the experts for comment; their comments were incorporated. Further piloting among six preschool teachers suggested the interview was sensitive enough to produce sufficient variability, with a range of scores from 12 to 34 and a standard deviation of 7 (scores between 0 and 86 being possible).

Because the PM-PCK Interview is a new measure, we assessed its internal reliability before examining construct validity. As a first measure, correlations were run between total scenario scores. Scores for Scenarios 1 and 2, each meant to reflect PCK across mathematics content areas, correlated strongly ($r = .616, p < 0.001$). Utilizing subscenario scores, Cronbach's alpha for the items within these two scenarios was a strong .76, representing an acceptable level of interitem reliability (according to Nunnally, 1978, the standard is .7). Interviews were coded live, and 15 (68%) were coded simultaneously by two researchers. Interrater reliability was 92.8%; disagreements were settled by discussion and used an interview audiotape. Teachers' scores ranged from 6 to 32 points, with an average score of 21.6 and a standard deviation of 5.8.

THEORETICAL FRAMEWORK

The design of the interview was influenced by literature on mathematics education (e.g., Ball, 1988; Hiebert & Lefevre, 1986), preschool pedagogy (e.g., Bowman, Donovan, & Burns, 2001; Clements, Sarama, & DiBiase, 2004), cognitive development (e.g., Kamii & DeVries, 1978; Mix, Huttenlocher, & Levine, 2002), and mathematics content for very young children (e.g., Copley, 2010; Ginsburg et al., 2008). Two particular sets of ideas—one from the mathematics education literature and one that draws on recent ideas about preschool mathematics—were instrumental to the interview design. Specifically, teaching scenarios were used, as in the work by Ball and colleagues (e.g., Hill et al., 2004) to elicit that subset of mathematical knowledge that is particularly useful in teaching it. Simultaneously, work on relationships between concepts and procedures in mathematics (e.g., Hiebert & Lefevre, 1986) informed the sense of what such teaching ought to be in preschool classrooms. As a means to clarify the theoretical framework that guided the development of the interview, these ideas are detailed and discussed below.

Assessing Mathematical PCK: The Use of Teaching Scenarios

According to Lee Shulman (1986), who first identified pedagogical content knowledge (PCK) during his 1985 presidential address to the American Educational Research Association, PCK is "a knowledge of subject matter for teaching which consists of an understanding of how to represent specific subject matter topics and issues appropriate to the diverse abilities and interest of learners" (Shulman & Grosman, 1988, p. 9). In other words, PCK is not a single, isolated knowledge base, but the coalescence of three types of knowledge necessary for effective instruction:

knowledge of content, of teaching practice, and of student development. That is, PCK emphasizes (1) knowledge of which content ideas are most central and how they connect to one another (subject matter understanding), (2) appropriate examples and strategies for illustrating those concepts (teaching techniques for the subject matter), and (3) awareness of how those concepts develop in the thinking of novices with differing levels of experience (knowledge of the development of student understanding of the subject matter). See Figure 1 for a diagram illustrating how these three bodies of knowledge overlap to construct PCK.

The PM-PCK Interview borrows from Deborah Ball's 1988 innovation of situating questions within mathematics teaching scenarios (see also Hill et al., 2004; Study of Instructional Improvement, 2002). This represents a true departure from traditional measures of teacher math knowledge, which ask teachers to solve math problems, rather than address problems students are having with mathematics. Using teaching scenarios presents two advantages over using items solely about content knowledge: it contextualizes math questions so they are more like the kinds of dilemmas teachers encounter in the classroom, and relatedly, it allows the interview to assess PCK as it naturally occurs—in an integrated form.

Consider Scenario 1 of the PM-PCK Interview. To respond to the question "What kinds of mathematics do you see in this play?," a teacher must analyze and unpack the mathematical ideas embedded in the play scenario. For example, is patterning involved in Brittany's and Jacob's behaviors of connecting different shoeboxes with the differing sizes of the babies? Is there evidence of the mathematical concepts of sorting, matching, or counting in this scenario? The ability to analyze a play situation and identify its opportunities for "mathematization" relies explicitly on teachers' in-depth understanding of content knowledge (Clements et al., 2004).

Teachers' responses to the question of how they might "expand on the math in this play" tell the interviewer a great deal about both their knowledge of children's development within the content strand, and about their knowledge of strategies they can employ to extend children's thinking about the content. For example, when Jacob notes that there "aren't enough cribs," a

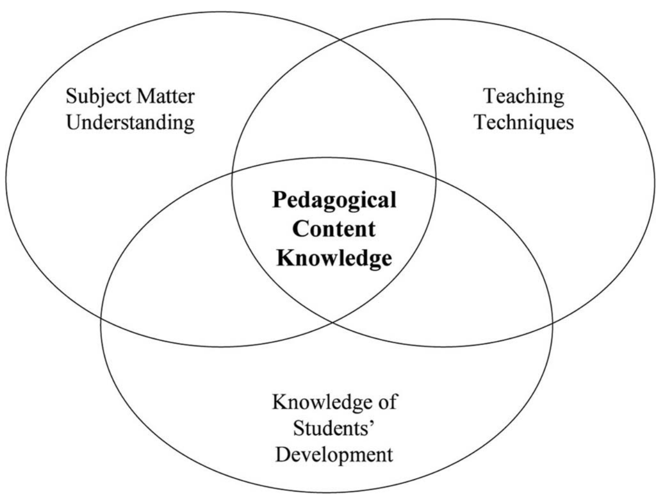

FIGURE 1 Pedagogical content knowledge (PCK).

teacher might ask him, "How do you know?," providing an opportunity for Jacob to talk about or demonstrate his thinking. This same teacher could also decide that, given Jacob's developmental stage, this is a great opportunity to connect his actions with a number story by asking him, "How many more cribs do you need?" and observing the methods Jacob uses to answer her question. In either case, the teacher's input relies heavily and simultaneously on an understanding of how ideas about number develop and her own awareness of techniques she can use to challenge children to move to the next stage of complexity in this content area. Knowledge of cognitive development in general terms will not suffice here: the knowledge of students the teacher reveals is domain specific, particular to early mathematics learning and development, and tailored to children's behavior in a typical play situation. By the same token, strategies the teacher chooses to help children explore the mathematics in their play are effective only when grounded in the mathematical content and incorporating a sense of its typical development. In this way, the teaching scenario provides a unique and powerful mechanism for tapping content knowledge, pedagogy, and understanding of student learners—as they relate to mathematics—simultaneously, acting as a better approximation of PCK than a purely content-related question can supply.

Good Preschool Math Teaching: The Role of Concepts and Procedures

Importantly, Ball's interview questions are also designed to assess teachers' ability to help elementary students maintain and enhance connections between procedures and concepts in their mathematical thinking. The idea that there are two distinct types of mathematical knowledge—procedural knowledge, which encompasses the forms, rules, and processes that make it possible to accomplish mathematical tasks, and conceptual knowledge, which embodies richly connected ideas about the mathematical relationships between things and actions—is an old one. U.S. mathematics education has a long history of emphasizing first one of these types of knowledge, and then the other, with little effective result (see, e.g., McLellan & Dewey, 1895; Thorndike, 1922; Wheeler, 1939). It was not until the early 1980s that Resnick and Ford (1981) suggested that rather than trying to determine their relative importance to math education, the relationships between concepts and procedures ought to be emphasized. In alignment with this idea, Ball's interview asks elementary mathematics teachers to generate real-world representations that can illustrate the usefulness and meaning of specific mathematics procedures.

For example, in one of Ball's (1988) interview scenarios, teachers were presented with the division statement $1\ \frac{3}{4}\ /\ \frac{1}{2}$ ("one and three quarters divided by one half"). Ball asked the teachers to "develop a representation—a story, a model, a picture, a real-world situation" of this division statement (p. 16). An appropriate model of this statement would demonstrate that it will answer the question, "How many halves are there in one and three quarters?" The effective teacher, Ball reasoned, would be able to describe a situation in which such a question is meaningful. As an example of a useful representation, Ball suggests using, "A recipe calls for $\frac{1}{2}$ cup of butter. How many batches can one make if one has $1\frac{3}{4}$ cups butter? Answer: $3\frac{1}{2}$ batches" (p. 16). Interview questions such as this one made clear which teachers had more of the kind of knowledge that would enable them to keep mathematical procedures meaningfully connected to the ideas they are meant to represent.

As in elementary mathematics, connections between concepts and procedures in preschool mathematical thinking are central. The meaning of the term *procedure*, however, is rather different (see, e.g., Hiebert & Lefevre, 1986). In the elementary-based literature, procedures are often algorithmic processes enacted upon symbols, such as “borrowing from the 10s column,” whereas in the preschool version of this analysis, procedures are physical (or mental) actions taken and enacted upon concrete objects. For example, a toddler implementing the “procedure” of placing a single spoon in each teacup can be said to be constructing a working concept of one-to-one correspondence (see Sinclair & Sinclair, 1986).

From this perspective, very young children’s mathematics concepts are completely dependent upon their procedures; mathematical knowledge is being created through action and has not yet been separated from the world of things. As Sinclair and Sinclair (1986) commented, “The young child cannot do without actual experience when logico-mathematical knowledge is in its beginnings” (p. 63). Quantitative development at this stage appears to be a process of increasing abstraction, in which links between actions (procedures) and the concepts they imply are first being formed (see, e.g., Kamii & DeVries, 1978). In preschool mathematics, procedural thinking is represented in the actions of young children upon their environment, including the language they use to describe the math they are thinking about (Clements, 2004; Copley, 2010).

If preschoolers can benefit from assistance in establishing and consolidating initial connections between their own actions, their budding mathematical ideas, and more generalized concepts, this provides an initial portrait of what good preschool mathematics teaching must be. Following the thinking of children as they interact with materials, recognizing the mathematical potential in their activities, and knowing how to comment on and extend their mathematics-related thinking all must be central. For these reasons, the PM-PCK Interview asks teachers to consider the preschool play scenario presented, identify specific math-related topics the children’s play addresses, make a comment to help the children think about/become more aware of the mathematics in their play, and ask a question that might encourage children to experiment with the mathematics in their play and extend their thinking. Instead of being asked to generate meaningful connections to mathematical procedures, as in Ball’s (1988) interview, teachers are asked to recognize the mathematics embedded in everyday activities and enhance its explicit abstraction and “mathematization.”

CONSTRUCT VALIDITY: PREDICTING TEACHING PRACTICE

To test the construct validity of the PM-PCK Interview, we first look at its predictive relationship with teaching practice. If the PM-PCK Interview is assessing good teacher knowledge, then good teacher practice should co-occur with high interview scores. Frequency of math-related language was used as a proxy for good teaching (see Measure and Procedures, below), because it has been shown to predict growth in preschool students’ mathematical knowledge (see, e.g., Ehrlich, 2007; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; McCray, 2008).

Teachers and Classrooms

Twenty-two Head Start teachers participated in this study. To be considered for the study, teachers had to work in classrooms in which English was not a second language for a majority of students, and they had to be the “head” teacher in their classrooms. The 22 classrooms were clustered within 16 sites. On average, teachers had 9 years of teaching experience. Five teachers had graduated from high school, four held associate's degrees, 10 held bachelor's degrees, and three teachers had a master's degree. Twelve teachers held a state certificate in early childhood education. On average, the teachers had taken a little more than two mathematics courses in college, whereas four reported none and one reported six. Mathematics inservice courses taken ranged from 0 (five teachers) to 10 (three teachers), with an average of 3.5 courses taken.

Measure and Procedures

Participating teachers' speech was digitally recorded and coded on one randomly selected day during the period from the beginning of January until the end of April. One hour of teacher speech was recorded and coded, always before noon, and included "circle time" (teacher-led large group) and the period immediately following; recording/coding continued until 60 minutes were captured. Activity after circle time consisted of free play, in which teachers supported or scaffolded children's activities in areas of their own choosing, and/or small-group activities, in which teachers met with two to eight children to guide them through a teacher-led set of activities. To facilitate recording, each teacher wore a wireless microphone, which broadcast a signal to be coded and recorded. Two researchers were present during recording to code teacher math-related language responses; a timer was utilized to prompt researchers to take turns coding in 10-minute intervals. Teachers were unaware of the mathematics focus of the study when the language sample was recorded and coded because the PM-PCK Interview was not conducted until the end of the school year. Coding categories from Klibanoff et al. (2006) were used to code teacher math-related language (see the appendix for category definitions). Because live coding of math-related language using this coding system had not been previously attempted, all language samples were recoded by the first author using audiotapes, yielding an interrater reliability score of 96.4%; disagreements were settled by discussion.

This study was particularly interested in the pedagogical settings in which teacher math-related language occurs. Specifically, it was theorized that because the PM-PCK Interview requires teachers to analyze free-play situations, it might be a better predictor of math-related language used in less-structured, rather than more-structured, settings. Accordingly, the study categorizes each math-related language instance as occurring either during or outside of "circle time"—a highly structured, large-group, teacher-led activity, commonly conducted in the morning. Both language measures—frequency of circle time math-related language (CTML) instances and frequency of non-circle time math-related language (NCTML) instances—are examined for relationships to PCK Interview scores.

Results

For the sample of 22 teachers, total frequency of math-related language ranged from 4 to 74 instances ($M = 30.86$, $SD = 20.65$). CTML ranged from 0 to 50 instances ($M = 18.95$, $SD = 16.01$), and NCTML ranged from 0 to 34 instances ($M = 11.91$, $SD = 9.65$). Circle and non-circle time frequencies did not correlate with one another ($r = .248$, $p = .133$, *ns*); that is, frequency of math-related language used during circle time did not significantly predict non-circle time frequency, and vice versa. Circle time ranged in length from almost 6 minutes to nearly 42 minutes ($M = 20.95$, $SD = 11.87$). As might be expected, teachers with longer circle times used more math-related language during that time ($r = .673$, $p < .000$). This relationship did not hold true for non-circle time, however ($r = -.144$, p = .261, *ns*); that is, teachers who spent more of the 60 minutes outside of circle time did not also tend to have more non-circle time math-related language. Frequency of math-related language, then, cannot be interpreted as simply a measure of time on task.

Two 2-level hierarchical linear models (HLMs) were run to determine whether PM-PCK Interview scores were related to either CTML or NCTML. HLM is generally considered appropriate whenever data has a nested structure, as it does in this study, because child outcomes are clustered by teacher and teachers are clustered by program sites. Specifically, the hierarchical structure of HLM allows examination of the effects of program site and how they interact with PCK. That is, we can determine whether and how site characteristics affect relationships between teacher PCK and teacher language.

Because this analysis assesses the relationship between the PCK Interview and teaching practices, the models do not include a level for child variables. Instead, teacher variables (which, in this case, include outcome and predictor variables) occur at Level 1 of each model, whereas Level 2 represents the clustering of teachers within program sites.

Level-1 Model

$$\text{CTML} = \beta_0 + r$$

Level-2 Model

$$\beta_0 = \gamma_{00} + \text{u}_0$$

When combined, these two levels yield a single model:

$$\text{CTML} = \gamma_{00} + \text{u}_0 + r$$

Preliminary analysis of the fully unconditional models (see Raudenbush & Bryk, 2002, p. 24) for math-related language during circle time (as above) and outside of circle time indicates that each differs significantly at the classroom level (CTML $t = 5.58$, $p < .000$; NCTML $t = 6.43$, $p < .000$), but neither differs significantly by program site (CTML $\chi^2 = 23.84$, $p = 0.07$, *ns*; NCTML $\chi^2 = 14.25$, $p > .500$, *ns*). In other words, there is significant variance in math-related language between teachers that is not due to the program site of which their classroom is a part.

To use HLM to assess the contribution of other variables, they are entered at the appropriate level. As an example, see the model for analyzing the relationship between the interview (INT) and CTML below.

Level-1 Model

$$\text{CTML} = \beta_0 + \beta_1 * (\text{INT}) + r$$

Level-2 Model

$$\beta_0 = \gamma_{00} + u_0$$

$$\beta_1 = \gamma_{10}$$

Results indicate that Interview scores significantly and positively predict math-related language during circle time and outside of circle time. The interview is a stronger predictor of math-related language during circle time than outside of it (see Table 1), because one Interview point is associated with only 3.7 instances of math-related language outside of circle time, but 7.75 instances during circle time.

CONSTRUCT VALIDITY: PREDICTING CHILDREN'S LEARNING

The construct validity of the PM-PCK Interview also was examined by testing its predictive relationship to children's mathematics learning outcomes. If the PM-PCK Interview is successful in assessing good teacher knowledge, then good child learning outcomes should co-occur with high interview scores. To assess children's learning over the course of the school year, the *Test of Early Mathematics Ability*, 3rd ed. (TEMA-3; Ginsburg & Baroody, 2003) was administered to children in the interviewed teachers' classrooms, once in the fall and once in the spring (see Measure and Procedures, below). Children also were administered an assessment of receptive language to camouflage the math focus of the study.

Children

One hundred and thirteen children were assessed at Time 1 (T1) in the 22 classrooms that participated, or an average of 5.1 children per classroom. These children averaged 4 years and 4 months of age at first testing, and 61 of them, or 53.9%, were female. One hundred and two children, or 90%, came from homes in which only English was spoken, whereas another 11 had been exposed to English and at least one other language in their home environment. Eighty-eight children, or 78%, were African American, 23 were of Latino origin, and 2 were identified as Asian/Pacific

TABLE 1
Results of Two-Level Hierarchical Linear Modeling to Evaluate Interview Scores as Predictors of Math-Related Language

Fixed Effect	*Coefficient*	*Standard Error*	*T-ratio*	*df*	*p Value*
Predicting Circle Time Math-Related Language					
For Intercept 1, β_0	20.365740	3.117610	6.532	15	0.000
For INTTOT slope, β_1	7.748238	2.915856	2.657	19	0.016
Predicting Non-Circle Time Math-Related Language					
For Intercept 1, β_0	11.909754	1.589447	7.493	15	0.000
For INTTOT slope, β_1	3.668737	1.626268	2.256	19	0.036

Note. INTTOT = Interview Scores.

Islander. Of these 113 children, 91 (80.5%) had mathematics assessment scores at two time points (others having moved away or dropped out of their class). Only these children are included in the analysis, resulting in an average of 4.1 children per classroom. The missing participants were demographically similar to the entire sample, and at Time 1, scored very similarly on the TEMA-3; specifically, the missing participants had a mean score of 89.91 with a standard deviation of 13.93, while the remaining sample had a mean score of 85.11 and a standard deviation of 12.43 (see Measure and Procedures, below).

Measure and Procedures

The TEMA-3 (Ginsburg & Baroody, 2003) is a standardized, norm-referenced instrument designed to assess informal (nonschool taught) and formal math achievements among 3- to 9-year-old children. TEMA-3 has two parallel forms with established test–retest reliability; children were randomly assigned Form A or Form B at T1 and received the alternate form at T2. Testing took place in a quiet setting near the child's classroom, often at a small table in the hallway just outside the room. Assessments were conducted by graduate students in child development.

Results

At T1, the 91 children scored between 65 and 132, with a mean score of 85.11 and a standard deviation of 12.43. This mean score is nearly one standard deviation below the population average of 100, indicating the sample children were doing noticeably worse in mathematics than their same-age peers nationally. Although discouraging, these scores seem reasonable, because the Head Start population necessarily comes from homes with fewer economic resources than are average for U.S. households. At T2, scores had not changed that much overall. The children scored between 62 and 124, with a mean of 86.11 and a standard deviation of 14.74. The average change in score was an increase of one point. The standard deviation in change scores for this sample was 9.91.

A three-level HLM (children at Level 1, teachers at Level 2, and program sites at Level 3) was used to examine the relationship between PCK Interview scores and changes in child scores from T1 to T2 (see Table 2). Results of the fully unconditional model for change on the TEMA-3 (TEMDIF) indicate that 19.6% of the variance in children's change scores can be attributed to differences at the classroom level. The fully unconditional model also indicates that this variance is significant, $\chi^2 = 25.86$ with 6 *df*s ($p < 0.000$), but variance between sites is not, $\chi^2 = 24.31$ with 15 *df*s ($p = .06$, *ns*). In other words, teachers (or something at the classroom level) appear to be having more of an impact upon children's mathematics learning than program sites.

To evaluate whether teacher PCK for preschool mathematics as measured by the PM-PCK Interview can explain some of this classroom-level variance, interview scores were entered as predictors at Level-2 (INT), resulting in the following model:

Level-1

$$\text{TEMDIF} = \pi 0 + e$$

Level-2

$$\pi 0 = \beta 00 + \beta 01(\mathrm{INT}) + r0$$

Level-3

$$\beta 00 = \gamma 000 + \mathrm{u}00$$

$$\beta 01 = \gamma 010$$

When interview scores are entered as a predictor, they significantly and positively predict gains in child outcomes; that is, the higher the teacher's PM-PCK score, the greater the gains children in her classroom made from T1 to T2 (see Table 2). Specifically, one point on the PCK Interview predicts 2.3 points of gain on the TEMA-3.

DISCUSSION

This study has several important limitations. These findings cannot be interpreted as demonstrating that better PCK causes better child outcomes. Because the findings are associational, it is possible that some unmeasured factor, such as teacher attitude toward mathematics, is the real engine behind children's learning. Controlled randomized design is required to ascertain cause. Further, the PCK Interview is limited in scope, because it only directly assesses teacher knowledge used in free-play settings. Although the teacher interview scores effectively predicted the use of math-related language regardless of pedagogical setting, the interview does not address planning, curriculum implementation, or child assessment, all of which have important implications for teaching preschool math.

Nevertheless, there is a clear association between the PM-PCK Interview and the two measures of construct validity. A one-point increase in interview score is associated with almost eight additional instances of math-related language during circle time, and almost four additional instances of math-related language outside of circle time, indicating a relationship between the construct measured by the interview and teaching practices that are associated with enhanced math learning. Given standard deviations of approximately 16 (circle time) and 10 (noncircle

TABLE 2
Results of Three-Level Hierarchical Linear Modeling to Evaluate Interview Scores as Predictors of Child Gains on Test of Early Mathematics Ability (3rd ed.)

Fixed Effect	*Coefficient*	*Standard Error*	*T-ratio*	*df*	*p Value*
For Intercept 1, π_0					
Intercept 2, β_{00}	0.943076	0.924464	1.020	15	0.324
INTTOT, β_{01}	2.344966	0.938801	2.498	19	0.022

Note. INTTOT = Interview Scores.

time) language instances, a two- to three-point difference on the interview is associated with a quite meaningful difference in teaching practice.

Perhaps more important, a one-point increase in interview score is also associated with 2.3 points of additional gain on the TEMA-3 for students over the course of the school year. That is, higher scores on the Interview predict not only increased use of math-related language by the teacher, but also greater gains in math achievement by his students from fall to spring. At least to some extent, the interview has transferred the important work of elementary mathematics education researchers (such as Hill et al., 2005) to a preschool setting by providing a measure of mathematical content knowledge that is significantly associated with good teaching and children's learning.

Association With Teacher Math-Related Language

It is surprising that the Interview has a more significant and stronger relationship to math-related language delivered during circle time than outside of it, because the Interview scenarios are focused on free-play interactions. It may be that circle time is simply the arena in which teachers are more likely to address math overall, so their level of PCK is more evident in their teaching practice in this setting. Given the recent emphases in early childhood on literacy and social-emotional development (Bowman et al., 2001), it is also likely that few preschool teachers, even those with stronger PCK, purposefully use free-play settings as a time to help children think mathematically. The math-related language observation tool may be picking up the general association many people have between mathematics and teacher-directed, large-group activities; that is, teachers may be more likely to talk about math when they are "teaching" the whole group than when they are working with a single child. Regardless, the fact that the PM-PCK Interview can predict math teaching behaviors in any setting suggests thoroughness and perhaps durability in its portrayal of the PCK construct.

Association With Child Outcomes

Relationships between the interview and child outcomes, though significant, are relatively small in size. According to findings, it would take an increase of four points on the PCK Interview to generate a difference in TEMA-3 scores that exceeds the 9.24 point significance marker set by the assessment's designers (see Ginsburg & Baroody, 2003). This four-point increase in interview score is less than its standard deviation of 5.8 points, but also a large chunk of the overall range of found scores, which was 26, with a high score of 32. This finding may be influenced by the fact that the overall quality of this sample of teachers' responses to the interview was not very high.

As an example, when presented with Scenario 1, in which children place babies in shoebox "cribs" of different sizes, teachers were quick to see the mathematics of measurement but rarely saw that the cribs could elicit thinking about shape and spatial relationships. When asked to comment on the block play depicted in Scenario 2, teachers noted the relevance of shape but very seldom mentioned the classification of blocks or the use of counting to build walls of the same length. It is possible to score as many as 86 points on the interview, or more than $2\frac{1}{2}$ times the highest score achieved by these teachers. The PCK of the study's teachers appears to be supporting the mathematical learning that does occur in these classrooms; that does not mean, however, that it could not be radically improved. And clearly, because average TEMA-3 scores for the sample did not show improvement, the teaching is not adequate to help this group of Head Start children “catch up” to their peers. An increase of at least four points in PCK Interview score might not be an unreasonable goal, despite the high score of 32 achieved here.

Need for Further Research

It would be a worthwhile exercise to try to expand the PM-PCK Interview to include teacher knowledge that is used outside the free-play setting, assessing the contribution to teaching that such knowledge makes. It might also be useful to create and validate additional free-play scenarios. Two more scenarios that yield similar results could function as “version B,” and would make it possible to use the PM-PCK Interview as part of an intervention study. Control and intervention teachers could be randomly assigned to one version at T1 and the other at T2, allowing examination of the interview as a measure of change in teacher knowledge. Development of the additional scenarios would require a similar process of face validity analysis and piloting to that described herein, followed by comparison with the existing scenarios.

CONCLUSIONS

The work described here suggests that PCK for preschool mathematics requires an understanding of the foundational concepts of mathematical content, combined with the skill to closely observe children’s play, discern their likely thinking, and provide language that points out embedded mathematics. Effective preschool teachers must recognize the importance and complexities of very basic experiences of size, shape, and quantity, and be prepared to label and connect such experiences for children.

The interview method—having teachers solve math teaching problems rather than math problems—demonstrated its effectiveness for measuring content knowledge with a meaningful relationship to teaching, regardless of the grade level of the teaching involved. That is, Ball’s (1988) innovation of having teachers respond to a real teaching situation successfully detected qualities among teachers that contribute to good teaching, even when transferred to a preschool setting. This substantiates the usefulness of the PCK concept for evaluating and understanding teaching across the grades, and more pointedly, it makes clear the ability of a particular assessment mechanism—the teaching scenario—to illuminate the kinds of teacher thinking that really matter for student learning.

This analysis also supports the idea that, just as establishing connections between procedures and concepts is crucial to meaningful elementary level mathematics education, building the initial connections between concepts and procedures is critical to the mathematical education of very young children. It suggests that in every setting, preschool teachers ought to promote these linkages, exercising pedagogy based on this understanding about the nature of mathematical content. Early childhood teachers need the type of knowledge that will help them promote such concept–procedure connections, whether during circle time or free play. For preschool mathematics, it appears that at least a significant proportion of the pedagogical content knowledge needed can be usefully described as teachers' ability to help children see and understand mathematics in the world around them.

Finally, the PM-PCK Interview provides a new means of assessing teacher knowledge for teaching preschool mathematics. Too often, the importance of good content knowledge for preschool teachers is minimized or ignored. Recent efforts in early math have helped create awareness that its teaching is critical and complex. Further studies that examine relationships between teachers' knowledge of foundational mathematics and the effectiveness of their teaching may provide ongoing contributions to a more precise and useful picture of what early math teaching ought to be.

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APPENDIX

MATH-RELATED LANGUAGE CODING SYSTEM FROM KLIBANOFF ET AL. (2006)

1. Counting: encompasses reciting counting words and counting objects in sets.
2. Cardinality: involves stating (or asking for) the number of things in a set without counting them. If cardinality is used to reinforce counting, it is coded as a separate instance (e.g., "One, two, three. There are 3 books" would be coded as two instances, one of counting and one of cardinality).
3. Equivalence: encompasses statements describing a quantitative match, either of number or of amount, between two or more entities. These include (1) one-to-one mapping (e.g., each child gets one cracker), (2) one-to-many mapping (e.g., each group has four children), and (3) stating that two amounts or sets are the same.
4. Nonequivalence: encompasses statements of two or more entities being unequal, whether referring to (1) unspecified amounts (e.g., "Who has the most?"), (2) one amount specified and the other(s) unspecified (e.g., "Oh no, you have more than 12 teeth"), and (3) all

relevant amounts specified (e.g., "Seven people said yes, ten people said no. Did more people say yes or say no?").

5. Number symbols: coded if utterances include instances in which a teacher labels a written number symbol, or asks a child to identify, write, or find a number symbol (e.g., "3" in a stack of cards with printed numbers).
6. Conventional nominatives: numbers used as labels for things or dates.
7. Ordering: instances of referral to a sequence with explicit reference to more than one entity or set. Note that reciting a list of number words in order would not be coded as ordering but rather as counting.
8. Calculation: includes cases in which a teacher performs a calculation or asks a child to solve a calculation problem.
9. Placeholding: encompasses any input that refers to place value: ones, tens, hundreds, etc., including, but not limited to, the decomposition of (at least) two-digit numbers.